RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE **4722/01**

MATHEMATICS

Core Mathematics 2

THURSDAY 15 MAY 2008 Morning

Time: 1 hour 30 minutes

Additional materials: Answer Booklet (8 pages)
List of Formulae (MF1)

INSTRUCTIONS TO CANDIDATES

- Write your name in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- **You are reminded of the need for clear presentation in your answers.**

This document consists of **4** printed pages.

[Turn over

1 Find and simplify the first three terms in the expansion of $(2-3x)^6$ in ascending powers of x. [4]

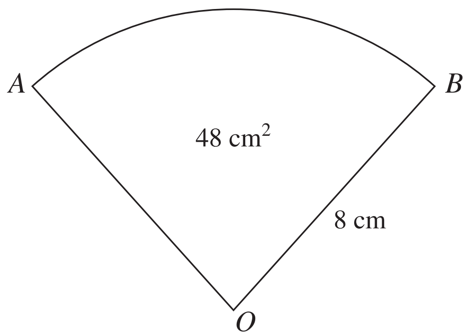
2 A sequence $u_1, u_2, u_3, \ldots$ is defined by

$$u_1 = 3 \quad \text{and} \quad u_{n+1} = 1 - \frac{1}{u_n} \text{ for } n \geqslant 1.$$

(i) Write down the values of u_2, u_3 and u_4. [3]

(ii) Describe the behaviour of the sequence. [1]

3



The diagram shows a sector AOB of a circle with centre O and radius 8 cm. The area of the sector is 48 cm^2.

(i) Find angle AOB, giving your answer in radians. [2]

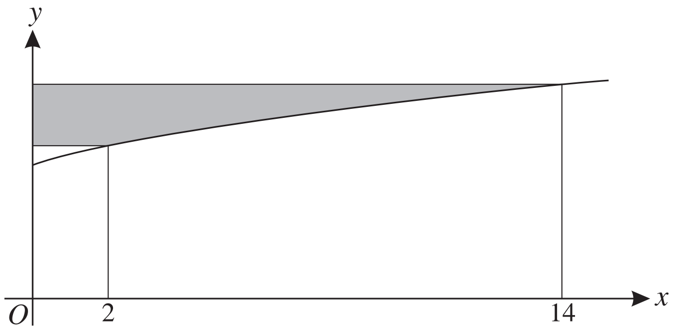
(ii) Find the area of the segment bounded by the arc AB and the chord AB. [3]

4 The cubic polynomial $ax^3 - 4x^2 - 7ax + 12$ is denoted by $\text{f}(x)$.

(i) Given that $(x-3)$ is a factor of $\text{f}(x)$, find the value of the constant a. [3]

(ii) Using this value of a, find the remainder when $\text{f}(x)$ is divided by $(x+2)$. [2]

5



The diagram shows the curve $y = 3 + \sqrt{x + 2}$.

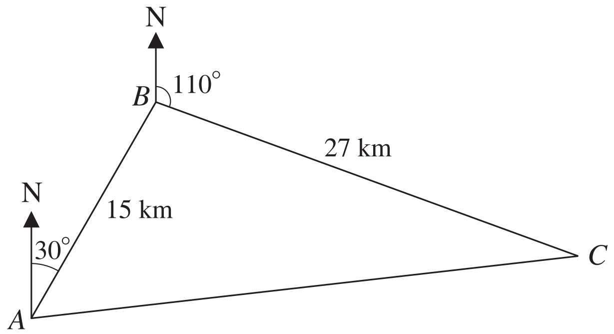
The shaded region is bounded by the curve, the y-axis, and two lines parallel to the x-axis which meet the curve where $x = 2$ and $x = 14$.

(i) Show that the area of the shaded region is given by

$$\int_5^7 (y^2 - 6y + 7)\,dy. \qquad [3]$$

(ii) Hence find the exact area of the shaded region. [4]

6



In the diagram, a lifeboat station is at point A. A distress call is received and the lifeboat travels 15 km on a bearing of 030° to point B. A second call is received and the lifeboat then travels 27 km on a bearing of 110° to arrive at point C. The lifeboat then travels back to the station at A.

(i) Show that angle ABC is 100°. [1]

(ii) Find the distance that the lifeboat has to travel to get from C back to A. [2]

(iii) Find the bearing on which the lifeboat has to travel to get from C to A. [4]

7 **(a)** Find $\int x^3(x^2 - x + 5)\,dx$. [4]

(b) **(i)** Find $\int 18x^{-4}\,dx$. [2]

(ii) Hence evaluate $\int_2^{\infty} 18x^{-4}\,dx$. [2]

8 **(i)** Sketch the curve $y = 2 \times 3^x$, stating the coordinates of any intersections with the axes. [3]

(ii) The curve $y = 2 \times 3^x$ intersects the curve $y = 8^x$ at the point P. Show that the x-coordinate of P may be written as

$$\frac{1}{3 - \log_2 3}.$$ [5]

9 **(a)** **(i)** Show that the equation

$$2\sin x \tan x - 5 = \cos x$$

can be expressed in the form

$$3\cos^2 x + 5\cos x - 2 = 0.$$ [3]

(ii) Hence solve the equation

$$2\sin x \tan x - 5 = \cos x,$$

giving all values of x, in radians, for $0 \leqslant x \leqslant 2\pi$. [4]

(b) Use the trapezium rule, with four strips each of width 0.25, to find an approximate value for

$$\int_0^1 \cos x \, dx,$$

where x is in radians. Give your answer correct to 3 significant figures. [4]

10 Jamie is training for a triathlon, which involves swimming, running and cycling.

- On Day 1, he swims 2 km and then swims the same distance on each subsequent day.
- On Day 1, he runs 2 km and, on each subsequent day, he runs 0.5 km further than on the previous day. (Thus he runs 2.5 km on Day 2, 3 km on Day 3, and so on.)
- On Day 1 he cycles 2 km and, on each subsequent day, he cycles a distance 10% further than on the previous day.

(i) Find how far Jamie runs on Day 15. [2]

(ii) Verify that the distance cycled in a day first exceeds 12 km on Day 20. [3]

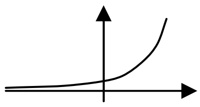
(iii) Find the day on which the total distance cycled, up to and including that day, first exceeds 200 km. [4]

(iv) Find the total distance travelled, by swimming, running and cycling, up to and including Day 30. [4]

4722 Core Mathematics 2

1		$(2 - 3x)^6 = 2^6 + 6.2^5.(-3x) + 15.2^4.(-3x)^2$	**M1**	Attempt (at least) first two terms - product of binomial coefficient and powers of 2 and (-)3x
		$= 64 - 576x + 2160x^2$	**A1**	Obtain $64 - 576x$
			M1	Attempt third term - binomial coefficient and powers of 2 and (-)3x
			A1	Obtain $2160x^2$
	OR			
			M1	Attempt expansion involving all 6 brackets
			A1	Obtain 64
			A1	Obtain $-576x$
			A1	Obtain $2160x^2$
	SR	if the expansion is attempted in descending order, and the required terms are never seen, then **B1 B1 B1** for $4860x^4$, $-2916x^5$, $729x^6$		
			4	
2	**(i)**	$u_2 = {}^2/_3$	**B1**	Obtain correct u_2
		$u_3 = {}^{-1}/_2$	**B1**√	Obtain correct u_3 from their u_2
		$u_4 = 3$	**B1**√	Obtain correct u_4 from their u_3
			3	
	(ii)	sequence is periodic / cyclic / repeating	**B1**	Any equivalent comment
			1	
3	**(i)**	$\frac{1}{2} \times 8^2 \times \theta = 48$	**M1**	State or imply $(\frac{1}{2})\ 8^2\theta = 48$
		Hence $\theta = 1.5$ radians	**A1**	Obtain $\theta = 1.5$ (or 0.477π), or equiv
			2	
	(ii)	area $= 48 - \frac{1}{2} \times 8^2 \times \sin 1.5$	**M1***	Attempt area of Δ using $(\frac{1}{2})\ 8^2 \sin \theta$
		$= 48 - 31.9$	**M1d***	Attempt 48 – area of Δ
		$= 16.1$	**A1**	Obtain 16.1 cm^2
			3	
4	**(i)**	$f(3) = 27a - 36 - 21a + 12 = 0$	**M1***	Attempt f(3)
		$6a = 24$	**M1d***	Equate attempt at f(3) to 0 and attempt to solve
		$a = 4$	**A1**	Obtain $a = 4$
	OR			
			M1*	Attempt complete division / matching coeffs
			M1d*	Equate remainder to 0
			A1	Obtain $a = 4$
			3	
	(ii)	$f(-2) = -32 - 16 + 56 + 12$	**M1**	Attempt f(-2)
		$= 20$	**A1**√	Obtain 20 (or $6a - 4$, following their a)
			2	

5 (i)	$\int x\mathrm{d}y = \int ((y-3)^2 - 2)\mathrm{d}y$	**B1**	Show $x = y^2 - 6y + 7$ convincingly
	$= \int (y^2 - 6y + 7)\mathrm{d}y$ **A.G.**	**B1**	State or imply that required area = $\int x\mathrm{d}y$
	$3 + \sqrt{(2+2)} = 5$, $3 + \sqrt{(14+2)} = 7$	**B1**	Use x = 2, 14 to show new limits of y = 5, 7
		3	
(ii)	$\left[\frac{1}{3}y^3 - 3y^2 + 7y\right]_5^7$	**M1**	Integration attempt, with at least one term correct
	$= (^{343}/_3 - 147 + 49) - (^{125}/_3 - 75 + 35)$	**A1**	All three terms correct
	$= 16^1/_3 - 1^2/_3$	**M1**	Attempt F(7) – F(5)
	$= 14\ ^2/_3$	**A1**	Obtain $14\ ^2/_3$, or exact equiv
		4	
6 (i)	$ABC = 360 - (150 + 110) = 100^\circ$ **A.G.**	**B1**	Show convincingly that angle ABC is 100°
		1	
(ii)	$CA^2 = 15^2 + 27^2 - 2\times15\times27\times\cos100^0$	**M1**	Attempt use of correct cosine rule
	= 1094.655…		
	CA = 33.1	**A1**	Obtain 33.1 km
		2	
(iii)	$\frac{\sin C}{15} = \frac{\sin 100}{33.1}$ or $\frac{\sin A}{27} = \frac{\sin 100}{33.1}$	**M1**	Attempt use of sine rule to find angle C or A (or equiv using cosine rule)
		A1√	Correct unsimplified eqn, following their CA
	$C = 26.5^\circ$ $A = 53.5^\circ$	**A1**	Obtain $C = 26.5^\circ$ or $A = 53.5^\circ$ (allow 53.4°)
	Hence bearing is 263°	**A1√**	Obtain 263 or 264 (or 290° – their angle C / 210 + their angle A)
		4	
7 (a)	$\int (x^5 - x^4 + 5x^3)\mathrm{d}x$	**M1**	Expand brackets and attempt integration, or other valid integration attempt
	$= \frac{1}{6}x^6 - \frac{1}{5}x^5 + \frac{5}{4}x^4\ (+c)$	**A1**	Obtain at least one correct term
		A1	Obtain a fully correct expression
		B1	For + c, and no $\int$ or dx (can be given in **(b)(i)** if not given here)
		4	
(b) (i)	$-6x^{-3}$ $(+c)$	**M1**	Obtain integral of the form kx^{-3}
		A1	Obtain $-6x^{-3}$ $(+c)$
		2	
(ii)	$\left[-6x^{-3}\right]_2^\infty$	**B1***	State or imply that F(∞) = 0 (for kx^n, n -1)
	= ¾	**B1d***	Obtain ¾ (or equiv)
		2	

8 (i)		**M1**	Attempt sketch of exponential graph (1st quad) - if seen in 2nd quad must be approx correct
		A1	Correct graph in both quadrants
		B1	State or imply (0, 2) only
		3	
(ii)	$8^x = 2 \times 3^x$		
	$\log_2 8^x = \log_2 (2 \times 3^x)$	**M1**	Form equation in x and take logs (to any consistent base, or no base) – could use $\log_8$
	$x\log_2 8 = \log_2 2 + x\log_2 3$	**M1**	Use $\log a^b = b \log a$
		M1	Use $\log ab = \log a + \log b$,or equiv with $\log {}^a/_b$
	$3x = 1 + x\log_2 3$	**M1**	Use $\log_2 8 = 3$
	$x(3 - \log_2 3) = 1$, hence $x = \dfrac{1}{3-\log_2 3}$ **A.G.**	**A1**	Show given answer correctly
OR	$8^x = 2 \times 3^x$		
	$2^{3x} = 2 \times 3^x$	**M1**	Use $8^x = 2^{3x}$
	$2^{(3x-1)} = 3^x$	**M1**	Attempt to rearrange equation to $2^k = 3^x$
	$\log_2 2^{(3x-1)} = \log_2 3^x$	**M1**	Take logs (to any base)
	$(3x-1)\log_2 2 = x\log_2 3$	**M1**	Use $\log a^b = b \log a$
	$x(3 - \log_2 3) = 1$, hence $x = \dfrac{1}{3-\log_2 3}$ **A.G.**	**A1**	Show given answer correctly
		5	
9 (a) (i)	$2\sin x.\dfrac{\sin x}{\cos x} - 5 = \cos x$	**M1**	Use $\tan x \equiv \dfrac{\sin x}{\cos x}$
	$2\sin^2 x - 5\cos x = \cos^2 x$		
	$2 - 2\cos^2 x - 5\cos x = \cos^2 x$	**M1**	Use $\sin^2 x \equiv 1 - \cos^2 x$
	$3\cos^2 x + 5\cos x - 2 = 0$	**A1**	Show given equation convincingly
		3	
(ii)	$(3\cos x - 1)(\cos x + 2) = 0$	**M1**	Attempt to solve quadratic in $\cos x$
	$\cos x = {}^1/_3$	**M1**	Attempt to find x from root(s) of quadratic
	$x = 1.23$ rad	**A1**	Obtain 1.23 rad or 70.5°
	$x = 5.05$ rad	**A1√**	Obtain 5.05 rad or 289° (or 2π / 360° - their solution)
			SR: B1 B1 for answer(s) only
		4	
(b)	0.5x0.25x{cos0+2(cos0.25+cos0.5+cos0.75)+cos1}	**M1**	Attempt y-coords for at least 4 of the correct 5 x-coords
		M1	Use correct trapezium rule, any h, for their y values to find area between $x = 0$ and $x = 1$
		M1	Correct h (soi) for their y values
	≈ 0.837	**A1**	Obtain 0.837
		4	

10 (i)	$u_{15} = 2 + 14 \times 0.5$	**M1**	Attempt use of $a + (n-1)d$
	$= 9$ km	**A1**	Obtain 9 km
		2	
(ii)	$u_{20} = 2 \times 1.1^{19} = 12.2$	**B1**	State, or imply, $r = 1.1$
		M1	Attempt u_{20}, using ar^{n-1}
	$u_{19} = 2 \times 1.1^{18} = 11.1$	**A1**	Obtain $u_{20} = 12.2$, and obtain $u_{19} = 11.1$
OR			
		B1	State, or imply, $r = 1.1$
		M1	Attempt to solve $ar^{n-1} = 12$
		A1	Obtain $n = 20$ (allow $n \geq 20$)
		3	
(iii)	$\frac{2(1.1^n - 1)}{(1.1 - 1)} > 200$	**B1**	State or imply $S_N = \frac{2(1.1^n - 1)}{(1.1 - 1)}$
	$1.1^n > 11$	**M1**	Link (any sign) their attempt at S_N (of a GP) to 200 and attempt to solve
	$n > \frac{\log 11}{\log 1.1}$	**A1**	Obtain 26, or 25.2 or better
	$n > 25.2$ ie Day 26	**A1**	Conclude $n = 26$ only, or equiv eg Day 26
		4	
(iv)	swum = 2 x 30 = 60 km	**B1**	Obtain 60 km, or 2 x 30km
	run = ½ x 30 x (4 + 29 x 0.5) = 277.5 km	**M1**	Attempt sum of AP, $d = 0.5$, $a = 2$, $n = 30$
	cycle = $\frac{2(1.1^{30} - 1)}{(1.1 - 1)}$ = 329.0 km	**M1**	Attempt sum of GP, $r = 1.1$, $a = 2$, $n = 30$
	total = 666 km	**A1**	Obtain 666 or 667 km
		4	